Geometry, Quarter 1, Unit 1.1

Developing Vocabulary Through Transformations

Overview

Number of instruction days: 7–9 (1 day = 53 minutes)

Content to Be Learned

- Understand that there are undefined notions of point, line, and distance along a line.
- Define terms to represent translations on a plane both verbally and visually: angles, perpendicular lines, parallel lines, and line segments.
- Represent transformations in a plane using transparencies and geometry software.
- Describe transformations as functions that take points as inputs and give other points as outputs.
- Compare transformations that may or may not preserve distance on or off the coordinate plane.

Mathematical Practices to Be Integrated

2 Reason abstractly and quantitatively.
- Understand and use units of measurement in transformations.

5 Use appropriate tools strategically.
- Represent transformations in a plane.

6 Attend to precision.
- Create and refine definitions using precise language.
- Use precision in measurements of transformations.

Essential Questions

- What is the precise language to describe a geometric transformation in a plane?
- How can a transformation be described with a function?
- How do numeric, symbolic, verbal, and graphic representations of transformations compare?
- How can you use transformations to preserve distance and to not preserve distance?
## Standards

### Common Core State Standards for Mathematical Content

#### Congruence (G-CO)

**G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**G-CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

### Common Core State Standards for Mathematical Practice

#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a
website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In Grade 4, students drew points, lines, line segments, rays, angles, and perpendicular and parallel lines. They identified these elements in two-dimensional figures. They also classified shapes by properties of their lines and angles. Fifth-grade students extended this knowledge to the coordinate plane. In Grade 8, students experimented with rigid motion through transformations that preserved distance and angle measure, and they described the effect of dilations on two-dimensional figures using coordinates.

Current Learning

In this unit, students learn precise geometry definitions for terms such as angles, perpendicular lines, parallel lines, and line segments based on the undefined notions of point, line, and distance along a line. Students also represent transformations and describe them as functions. They compare transformations that preserve distance with those that do not preserve distance. They experiment with transformations in the plane. This unit builds on students’ experience with rigid motion from earlier grades. Students learn that in geometry, translations move points a specified distance along a line parallel to a specified line and rotations move objects along a circular arc with a specified center through a specified angle. This work supports upcoming work to build fluency in triangle congruence, coordinate geometry, and constructions.

Future Learning

This unit’s work is foundational to the rest of Geometry. For example, students will use the definitions they learn in this unit in other units involving triangles, polygons, circles, and three-dimensional figures. Students will perform transformations of functions in the coordinate plane in Algebra II and Precalculus.
Additional Findings

The work in this unit is challenging for students because of the precise language and extensive vocabulary it requires. While students may have seen this unit’s vocabulary before, the terms are not used in everyday language and may be particularly difficult for English-language learners. It is important to clarify multiple meanings of words so that students understand that mathematics requires very precise and specific definitions and understanding of the terms. “High school students should learn multiple ways of expressing transformations . . . as well as function notation.” (Principles and Standards for School Mathematics, p. 43)

Assessment

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Demonstrate conceptual understanding of the precise definitions of angles, circles, perpendicular lines, parallel lines, and line segment.
- Analyze and compare transformations which may or may not preserve distance and angles.
- Use function notation to apply knowledge of transformations.
- Use multiple representation of transformation to demonstrate conceptual understanding.
Instruction

Learning Objectives

Students will be able to:

- Create, refine, and understand the definition of a line segment based on undefined terms of point and line.
- Create, refine, and understand the precise definition of an angle based on the undefined terms of point and line.
- Develop understanding about the concepts and attributes of parallel and perpendicular lines.
- Use concrete models and technology to perform and analyze transformations in a plane.
- Know a precise definition of circle as a distance around a circular arc.
- Symbolically represent transformations using function notation.
- Identify characteristics of transformations that preserve distance and angle measure with those that do not.
- Demonstrate and review knowledge of important concepts and procedures related to basic transformations.
Resources

*Geometry, Glencoe McGraw-Hill, 2010, Student/Teacher Editions*

- Section 1-1 (pp. 5 – 12)
- Section 1-2 (pp. 14 – 21)
- Section 1-3 (pp. 25 – 44)
- Section 1-5 (pp. 46 – 54)
- Sections 3-1 (pp. 171 – 176)
- Section 7-6 (pp. 505 – 511)
- Section 10-1 (pp. 683 – 691)
- Chapter 1 Resource Masters
- Chapter 3 Resource Masters
- Chapter 7 Resource Masters
- Study Notebook
  - 1-2 Linear Measure (pp. 5 - 6)
  - 1-4 Angle Measure (p. 9)
  - Chapter 3 Parallel and Perpendicular Lines (p. 39)
  - 7-6 Similarity Transformations (pp. 119 – 120)
- Real-World Problem Solving Graphic Novels Geometry (pp. 13 - 15)
- Interactive Classroom CD (PowerPoint Presentations)
- Teacher Works CD-ROM

Exam View Assessment Suite

*Transformations with Lists (ID: 10278).* Activity can be found at education.ti.com. See the Supplementary Unit Materials section of this binder for the student and teacher notes for this activity.

*Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.*
Materials

Rulers with centimeters and inches, protractors, white paper, TI-nspire graphing calculator, graphic organizers (optional)
Instructional Considerations

Key Vocabulary

No new vocabulary.

Planning for Effective Instructional Design and Delivery

Reinforced vocabulary from previous grades or units: angle, circle, transformation, parallel, perpendicular, and line segment.

Use nonlinguistic representations such as word walls, foldables, or graphic organizers to support the learning of vocabulary. Vocabulary support is essential to student success in Geometry. The undefined notions of point, line, and distance along a line are used to develop precise definitions for angle, perpendicular line, parallel lines, and line segment. Introducing tools and strategies for strong vocabulary development early in the course provides students with a structure upon which to scaffold their understanding of increasingly complex ideas as the course progresses. Use concrete, real-world examples to help students grasp abstract notions of these elements of Euclidean geometry. Students at a lower level of geometric development (e.g., Van Hiele model) may struggle with spatial visualization and may not be able to successfully interpret two-dimensional diagrams of three-dimensional abstract representations. Concrete models, such as those constructed with toothpicks and modeling clay or sliced pieces of paper, help struggling students with notions such as intersecting lines and planes.

Students come to this unit with prior experience with measuring length, using coordinate planes, and using the Pythagorean theorem. They also developed a strong set of measurement skills as a fluency skill. Additionally, they developed a strong set of measurement skills as a fluency skill in their previous work with geometric concepts. Capitalize on this prior knowledge and have students identify similarities and differences of one-dimensional number lines and two-dimensional coordinate systems to strengthen their comprehension of midpoint and distance formulas. Student’s continued work with distance along the line will help facilitate conceptual understanding of the meaning of line segment. Students can identify similarities and differences by using analogies, such as “absolute value is to the number line as the Pythagorean theorem is to the coordinate system.” They can also compare processes, such as finding the mean of a set of numbers versus finding the midpoint of a segment.

Students have had previous experience with conjecture using geometric reasoning. However, as students continue to explore a variety of geometric relationships, creating and testing examples, non-examples, and patterns, they will further develop their ability to conjecture and support their thinking in meaningful ways.

Section 1-1 can be used to activate student’s prior knowledge of the undefined notions of points, and lines in order to support the conceptual understanding of line segments and angles. In section 1-2, Omit example 6 as well as the segment construction activity as constructions will be explored in Unit 2.1. Similarly, maintain focus on the content standard G-CO.1 in section 1-4 with the development of the definition of angle being created and refined based on the undefined terms, excluding references to
classifying and constructing angles. Sections 1-5 and 3-1 support conceptual understanding of parallel and perpendicular lines and consequently should not be taught in their entirety. For instance, it is only necessary to teach the key concepts of parallel and skew lines and review example 1 in section 3-1, as angle relationships formed by parallel lines and transversals will be explored in a subsequent unit.

Conceptual understanding of the basic geometric structures is necessary to support the content of the units relating to transformations. They explore rigid motions which preserve distance and angles to those which don’t by using multiple representations. Students use these multiple representations to compare and describe transformations on and off the coordinate plane maintaining precision of language and measurement. They extend this knowledge to the definition of circles as a circle is created when a line segment (radius) is rotated 360 degrees around the specified center. This knowledge of rigid motion will reinforce the notion that circles are generated by moving an object along a circular arc with a specified center through a specified angle (CCSS Appendix A, p. 30).

When teaching translations, it is important to remember the vertical alignment of concepts among high school mathematics courses. Emphasize the function notation of translations \([x, y] \rightarrow (x + h, y + k)\] instead of using translation vectors, \(<h, k>\), which are used in the Glencoe McGraw-Hill Geometry textbook, since the function notation is what students will encounter on standardized tests.

Graphing, physical models, and technology (e.g. Transformation with Lists activity) provide the opportunity to use nonlinguistic representations as students investigate transformations. This strategy enables kinesthetic and visual learners to inductively draw conclusions based on their observations as they organize knowledge. The benefit of writing transformations using graphical representations is twofold: (1) it helps lay a solid foundation for students when they use matrices to perform rotations, and (2) it serves as a bridge to symbolic representation of transformations in higher mathematics courses.

To assist students with notetaking skills, have them create a foldable study organizer such as the one illustrated on page 4. Students can create and use a foldable to take notes, define terms, capture key concepts and ideas, and write illustrated examples. In this unit, there are numerous opportunities to differentiate instruction and scaffold support in the Study Notebook. For example, activate prior knowledge by using a KWL for parallel and perpendicular lines or skim lesson 4-7 to develop vocabulary necessary for exploration of congruence transformations.

As you assess students using the 5-minute check transparencies, a cues, questions, and advance organizers strategy is being used, since students are answering questions about content that is important. Some of the questions help students review prior knowledge, and these should be used at the beginning of a lesson; other questions, for use during and after the lesson, help students practice knowledge.
Notes