Triangle Congruence and Proofs

Overview

Number of instruction days: 8-10 (1 day = 53 minutes)

Content to Be Learned

- Understand how rigid motion relates to triangle congruence.
- Prove triangles to be congruent by ASA, SAS, and SSS using a variety of methods of proof both formal and informal.
- Prove triangle theorems, including:
  - Measures of interior angles of a triangle add up to 180°;
  - Base angles of isosceles triangles are congruent;
  - The segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length;
  - The medians of a triangle meet at a point.

Mathematical Practices to Be Integrated

2 Reason abstractly and quantitatively.
- Make sense of relationships among segments and angles in order to write informal and formal proofs.

3 Construct viable arguments and critique the reasoning of others.
- Use relationships among segments and angles, justify conclusions, communicate to others, and respond to arguments of others.
- Critique the arguments of others by comparing the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is.

6 Attend to precision.
- Utilize precise definitions in the construction of arguments and in discussions with others for a common understanding.
- Use precision in determining measurements of segments and angles.

7 Look for and make use of structure.
- Identify and use relationships in order to
construct viable arguments or proofs.

- Prove triangles to be congruent by ASA, SAS, and SSS using a variety of methods of proof both formal and informal.

**Essential Questions**

- How does rigid motion relate to triangle congruence?
- How can you prove two triangles are congruent?
- How do you prove or disprove conjectures using logical reasoning?
- How do you use informal and formal proofs to justify your thinking?
Common Core State Standards for Mathematical Content

Geometry

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Understand congruence in terms of rigid motions [Build on rigid motions as a familiar starting point for development of concept of geometric proof]

G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems [Focus on validity of underlying reasoning while using variety of ways of writing proofs]

G-CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Common Core State Standards for Mathematical Practice

2  Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3  Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about
data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see \( 7 \times 8 \) equals the well remembered \( 7 \times 5 + 7 \times 3 \), in preparation for learning about the distributive property. In the expression \( x^2 + 9x + 14 \), older students can see the 14 as \( 2 \times 7 \) and the 9 as \( 2 + 7 \). They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see \( 5 - 3(x - y)^2 \) as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers \( x \) and \( y \).

**Clarifying the Standards**

**Prior Learning**

Students were introduced to angles and measuring angles in Grade 4. In middle school, students used triangle relationships to solve problems, including determining when three angles or sides create a unique triangle, more than one triangle, or no triangle. In Grade 7, students reasoned about relationships among two-dimensional figures using scale drawings and informal geometric constructions. They described geometric figures and the relations between them and they drew geometric shapes with given conditions. In Grade 8, understanding congruence was a major cluster.
Students worked with congruence and similarity using physical models, transparencies, and geometry software. In their previous course work with geometric concepts, students have developed fluency in a strong set of measurement skills.

**Current Learning**

Conceptual understanding of triangle congruence is a fluency recommendation for students. Understanding triangle congruence is a foundation for continued success in Geometry. In this course, students formalize triangle relations they learned in middle school into theorems. Geometry students establish congruence between two triangles using rigid motion. Students investigate and formally prove, through a variety of methods, triangle congruence using the following postulates and theorems: SAS, ASA, and SSS. They extend their understanding to properties of isosceles and equilateral triangles and prove theorems about triangles, including that the measures of interior angles of a triangle add up to 180°, the base angles of isosceles triangles are congruent, the segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length, and the medians of a triangle meet at a point. Understanding congruence in terms of rigid motions and proving geometry theorems are major content as defined by the PARCC Model Frameworks for Mathematics.

**Future Learning**

This rigid motion unit will lay a foundation for geometric applications and extensions later in this course and in subsequent mathematics courses. A thorough knowledge of triangle congruence will prepare students to solve problems from physical situations using trigonometry, including the use of the Law of Sines, the Law of Cosines, and area formulas. Students will also encounter coordinate transformations using matrices in Precalculus.

**Additional Findings**

There are no additional findings for this unit.

**Assessment**

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.
The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Explain how triangle congruence theorems (ASA, SSS, and SAS) are related to rigid motion.
- Formally and informally prove triangles congruence (ASA, SSS, and SAS) using multiple methods of proof.
- Use logical reasoning to prove and apply triangle theorems including:
  - measures of interior angles of a triangle sum to 180°
  - base angles of isosceles triangles are congruent
  - the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length
  - the medians of a triangle meet at a point.

Instruction

Learning Objectives

Students will be able to:

- Prove the angle sum theorem and exterior angle theorem for triangles and use these theorems to solve real-world problems.
- Use formal and informal methods of proof, including flow, paragraph, and two-column, to show that two triangles are congruent (SSS, SAS, ASA).
- Use inductive reasoning to develop the properties of equilateral and isosceles triangles, and apply those properties to solve mathematical and real-world problems.
- Use coordinate geometry to prove triangle congruence.
- Use a variety of methods of construction, including compass and straightedge or patty paper, to investigate and prove attributes of medians of triangles.
- Review and demonstrate important concepts and procedures related to triangle congruence and proof.
Resources

*Geometry, Glencoe McGraw-Hill, 2010, Teacher/Student Editions*

- Section 4-2 (pp. 244-252)
- Explore 4-2 Geometry Lab: Angles of Triangles (p. 243)
- Sections 4-4 through 4-6 (pp. 262 – 291)
- Section 4-8 (pp. 301 - 307)
- Section 5-2 (pp. 333-341)
- Chapter 4 Resource Masters (pp. 15, 29, 41)
- Study Notebook
  - 4-2 Angles of Triangles (pp. 59 - 60)
- *Teaching Geometry with Manipulatives* (pp. 62-65, 68-70)
- Interactive Classroom CD (PowerPoint Presentations)
- Teacher Works CD-ROM

Exam View Assessment Suite

*Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.*

Materials

Graphic organizers, rulers, compass (optional), white paper, patty paper, graph paper, chart graph paper, TI-Nspire graphing calculator, dynamic geometry software

Instructional Considerations

Key Vocabulary

- auxiliary line
- median
- centroid
- SAS
- SSS
- ASA
Planning for Effective Instructional Design and Delivery

Reinforced vocabulary from previous grades or units: equilateral triangle, isosceles triangle, and angle sum theorem.

This unit affords many opportunities for hands-on student learning. In the following example students can use physical models (nonlinguistic representations) to conceptually anchor important ideas and theorems about triangles:

- Explore 4-2 Geometry Lab: Angles of Triangles, p. 243

Additional opportunities to differentiate instruction appear throughout the unit, including:

- Dinah Zike foldable for triangle congruence theorems, p. 234
- Section 4-2: Differentiated Instruction: Visual/Spatial Learner, p. 246
- Dinah Zike foldable for Chapter 5 study organizer, p. 320

Opportunities for including technology also appear throughout this unit. Students can use dynamic geometry software on the TI-Nspire to investigate the interior and exterior angle relationships of triangles (Section 4-2), properties of isosceles and equilateral triangles (Section 4-6), and medians and centroids (Section 5-2) of triangles.

Patty paper constructions can be used instead of or in addition to constructions with straightedge and compass. When using patty paper, make folds to construct the figure; the folds can be emphasized with pencil lines (don’t use a pen). The following instructions are for constructing some basic geometric figures such as lines, segments, perpendicular and parallel segments, angles, bisectors of lines and angles, and midpoints of lines.

- To construct line \( m \):
  
  Fold the patty paper in any direction, crease the paper, draw the line with a ruler if you choose, and label the line.

- To construct line segment AB:
  
  Construct a line as above, but do not draw it in or label it. Put points A and B on the line and connect the points to make the segment.

- To find the midpoint, C, of a segment:
  
  Construct a segment as above. Fold the paper so that A and B touch each other, and pinch the paper to find the midpoint. Label the point C.

- To construct the midsegment DE of a line:
Find a midpoint as above, but instead of pinching just at the midpoint, fold the paper when the points are touching. Draw the line and add points D and E to label the segment.

- To construct perpendicular lines:

Construct a midsegment as above and measure the angles to show that they are 90°. DE is perpendicular to AB.

- To construct parallel lines:

Follow the instructions for perpendicular lines. Make another line by folding line AB onto itself at some point other than the midpoint. Label that line FG. FG is perpendicular to AB, and since FG and DE are both perpendicular to AB, they are parallel to each other.

- To construct an angle, CAB:

Construct line segment AB above. Make another line that intersects AB at point A. Put point C on the new line. Extend line segments AC and AB to make them into rays.

- To construct a line, JK, perpendicular to another line p from a point not on p.

Make a line as above, label it p. Make a point J above p. Make a fold through point J so that line p is folded onto itself.

- To construct quadrilaterals of various types, use combinations of the above constructions.

In the previous unit, students established triangle congruence based on the analysis of rigid motion and construction. In this unit, they formalize their understanding relating rigid motion to triangle congruence and extend it to generalizations and logical reasoning proving statements are true. It is important to maintain a focus on the overarching goal of proving triangles congruent with constructions being integrated when appropriate to reinforce the objective. A variety of formats, both formal and informal, must be incorporated into the proofs. It is critical for students to develop deep conceptual understanding of triangle congruence as it is a fluency recommendation in PARCC.

As you assess students using the 5-minute check transparencies, a cues, questions, and advance organizers strategy is being used, since students are answering questions about content that is important. Some of the questions help students review prior knowledge, and these should be used at the beginning of a lesson; other questions, for use during and after the lesson, help students practice knowledge.