Using Coordinate Geometry to Prove Theorems and Solve Problems

Overview

Number of instruction days: 8-10 (1 day = 53 minutes)

Content to Be Learned

- Prove simple theorems using coordinate geometry.
- Prove the slope criteria for parallel and perpendicular lines.
- Solve problems involving parallel and perpendicular lines.
- Find the point on a line segment between two given points that divides the segment into a given ratio.
- Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles.
- Prove theorems about parallelograms.

Mathematical Practices to Be Integrated

1. Make sense of problems and persevere in solving them.
   - Understand the problem, develop a strategy for solving it, carry out the strategy, and analyze the results.
   - Solve problems on the coordinate plane involving perimeter of polygons and area of triangles and rectangles.

3. Construct viable arguments and critique the reasoning of others.
   - Create, develop, and present proofs.
   - Build logical progressions of statements to explore the truth of conjectures.
   - Recognize and use counterexamples.

4. Model with mathematics.
   - Use real-world situations in problems involving polygons.

6. Attend to precision.
   - Use precise definitions of geometric terms in proofs of theorems.
Essential Questions

- How is the distance formula related to the Pythagorean Theorem?
- Why are linear equations important in the study of proving geometric theorems algebraically?
- How do we use the slope criteria for parallel and perpendicular lines to solve geometric problems?
- What are the properties of parallelograms?

Standards

Common Core State Standards for Mathematical Content

Geometry

Expressing Geometric Properties with Equations [G-GPE]

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Prove geometric theorems [Focus on validity of underlying reasoning while using variety of ways of writing proofs]
G-CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students worked with a variety of measures in kindergarten through Grade 8, including length, area, volume, angle, surface area, and circumference. In Grade 5, students began plotting points on the coordinate plane. In Grade 8, students learned the Pythagorean Theorem and used it to find distances in the Cartesian coordinate system. In Algebra I, students explored the slopes of parallel and perpendicular lines and calculated linear equations of each. The algebraic techniques developed in Algebra I will be applied to the study of algebraic geometry.

Current Learning

Students prove the distance formula and relate it to the Pythagorean Theorem. They connect algebra to geometry in investigations on the coordinate plane. Geometric objects are analyzed using algebraic equations. This knowledge will be extended to proving basic geometric theorems in the Cartesian plane using algebra. Students make and verify conjectures about parallel and perpendicular lines using
coordinate proofs. Additionally, students use perimeter and area on the coordinate plane to solve problems. Using coordinates to prove simple geometric theorems algebraically is major content as defined by the PARCC Model Frameworks for Mathematics.

**Future Learning**

In Algebra II, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of conic sections. In Precalculus, students will extend their conceptual understanding of coordinate systems to the complex coordinate plane and analysis of vectors.

**Additional Findings**

Proofs can be challenging to students because of the need to justify reasoning to make meaning clear. Practice using a variety of techniques and perseverance help students to develop fluency in such reasoning. *(Adding It Up, p. 280).*

This content may be challenging for students since it requires them to justify reasoning in proof. A *Research Companion to Principles and Standards for School Mathematics* cites Thompson’s (1966) definition of mathematical reasoning as “purposeful inference, deduction, induction and association in the areas of quantity and structure.” (p. 228) *Adding It Up* describes adaptive reasoning as “the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions.” (p. 129)

**Assessment**

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.
• Use coordinate geometry to:
  
  prove the distance formula,
  
  prove a given figure is a rectangle,
  
  calculate perimeters of polygons,
  
  compute areas of triangles and rectangles.

• Prove lines are parallel or perpendicular using slope.

• Apply knowledge of parallel and perpendicular lines to solve real-world geometric problems.

• Precisely calculate the coordinates for a point on a line segment between two points when given a ratio partitioning the segment into two sections.

• Understand the connection between the Pythagorean Theorem and the distance formula.

• Prove the following theorems about parallelograms:
  
  opposite sides are congruent,
  
  opposite angles are congruent,
  
  diagonals of a parallelogram bisect each other,
  
  rectangles are parallelograms with congruent diagonals.

**Instruction**

**Learning Objectives**

Students will be able to

• Prove the distance formula.

• Apply coordinate geometry to solve problems involving perimeter and area of polygons.

• Prove that two lines are parallel or perpendicular using a variety of methods.

• Make conjectures about patterns among the slopes of parallel or perpendicular lines and apply this learning to real-life applications.
• Make and verify conjectures about the properties of parallelograms and use those properties to solve problems.

• Use a variety of methods to prove that a quadrilateral is a parallelogram and apply properties of parallelograms to problem situations.

• Make and verify conjectures about the properties of rectangles and apply these properties to solve problems.

• Make and verify conjectures about the properties of squares and rhombi to solve problems.

• Make and verify conjectures about proportional parts of segments between parallel lines using a variety of tools, including concrete models, and then use those conjectures to solve meaningful problems.

• Review and demonstrate knowledge of important concepts and procedures related to coordinate geometry.

Resources


• Section 1-6 (pp. 56 - 64)

• Sections 3-3 – 3-6 (pp. 186 – 222)

• Sections 6-2 – 6-5 (pp. 399 – 434)

• Section 7-4 (pp. 484 – 493)


• Chapter 1 Resources Masters (pp. 38 – 41)

• Chapter 3 Resources Masters (pp. 17 – 23, 30 – 34)

• Chapter 6 Resources Masters (pp. 11 – 36)

• Chapter 7 Resources Masters (pp. 24 - 29)

• Interactive Classroom CD (PowerPoint Presentations)

• Teacher Works CD-ROM

TI-Nspire Teacher Software
CCSS Lab 4: Geometry Lab: Proofs of Parallel and Perpendicular Lines (See Supplemental Materials section.)

Exam View Assessment Suite

Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.

Materials

Graph paper, straightedge, TI-Nspire graphing calculator

Instructional Considerations

Key Vocabulary

auxiliary line

coordinate proof

Planning for Effective Instructional Design and Delivery

Reinforced vocabulary taught in previous grades or units: slope, theorem, polygon, parallelogram, Pythagorean Theorem, distance formula, perpendicular and parallel.

In this unit, students extend their previous work with proofs by using coordinates to prove simple geometric theorems algebraically. They begin this exploration with a coordinate proof of the distance formula. There are numerous sources available for proofs of this theorem including an example in the Glencoe textbook on page 26. The emphasis in Section 1-6 should maintain focus on using the coordinate system to determine perimeter and area of polygons. Attention should be on the items using this similar approach. The chapter resource masters may also be used as a resource to differentiate content resources as needed.

Additional district resources for proofs of parallel and perpendicular lines are available in the Supplemental Unit Materials Section of this binder. They are also accessible online on the Glencoe McGraw-Hill Math Geometry Student and Teacher textbooks or on the following website: http://glencoe.mcgraw-hill.com/sites/dl/free/0078884845/634463/CCS_Geometry_se.pdf. To access the CCSS Supplements using the online textbooks select the CCSS icon on the homepage of the online textbook and then choose the corresponding lesson or lab to access supplementary Glencoe lessons identified in the resource section.

It is important to identify similarities and differences among different types of quadrilaterals and emphasize them—in particular, parallelograms. In each of Sections 6-2 through 6-5, graphic organizers of various forms of logic, such as Venn diagrams, logic chains, and flow charts, can be used to compare definitions and properties as students represent their knowledge.
Additionally, on page 388 at the beginning of Chapter 6, the foldable organizer has students use nonlinguistic representations. Using physical models while summarizing and taking notes helps students make sense of the attributes of different types of quadrilaterals, thereby organizing knowledge. To review the content and make important connections, students are using concept maps such as the one featured in the Study Notebook—Tie it Together: Quadrilaterals on page 105.

Journal writing is an appropriate place for using a summarizing and notetaking strategy. You can also ask students to respond to the Essential Questions at appropriate times throughout the lessons of this unit.