Algebra I, Quarter 1, Unit 1.3
Constructing, Interpreting, and Applying Linear Functions

Overview

Number of instruction days: 10–12
(1 day = 53 minutes)

Content to Be Learned

- Identify domain and range in terms of a function.
- Use function notation to evaluate and interpret the results of solving word problems.
- Interpret functions in terms of the context that relates the domain of a function to its graph.
- Recognize that sequences are functions.
- Write a function that describes a quantitative relationship by determining an explicit expression, a recursive process, or steps for calculation.
- Use arithmetic operations to combine functions.

Mathematical Practices to Be Integrated

2 Reason abstractly and quantitatively
- Using arithmetic operations, create functions that describe a relationship between two quantities.
- Evaluate functions when given the domain.
- Use different techniques to justify that a relationship is a function.

4 Modeling with Mathematics

- Construct and compare functions using multiple representations in terms of given context.
- Use a graphing calculator to relate the domain of a function to its graph.
- Use the recursive form of arithmetic sequences to model real-world situations.
- Write a function that describes a relationship between two quantities.

Essential Questions

- What do domain and range of a function represent in functions?
- Why is function notation important?
- What is the difference between a function and an equation?
- How is an arithmetic sequence related to a linear relationship?
- What type of dependency exists between the domain and the range of a function?
- How do you use a graphing calculator to interpret functions in terms of a context?
- How can you determine whether a sequence represents a function?
Common Core State Standards for Mathematical Content

Functions

Interpreting Functions

Understand the concept of a function and use function notation [Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences]

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

Interpret functions that arise in applications in terms of the context [Linear, exponential, and quadratic]

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.*

Building Functions

Build a function that models a relationship between two quantities [For F.BF.1, 2, linear, exponential, and quadratic]

F-BF.1 Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
2  **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4  **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**Clarifying the Standards**

**Prior Learning**

In Grade 8, students learned that a function is a rule that assigns each input to exactly one output. They learned how to graph a function using a set of ordered pairs, and they learned that functions can be presented in different ways, including graphically, numerically in tables, or by verbal description. Students also learned how to interpret a slope–intercept form equation \( y = mx+b \) and that this type of equation represents a linear function. They also learned to determine when a function is not linear.

**Current Learning**

Algebra I students learn function notation, \( f(x) \) and understand and use sequences as functions. Students learn that functions represent real-world situations with specific restrictions. They learn how to evaluate, interpret, and make predictions from a given context. They also learn how to determine from a table whether a function exists, and then identify the domains and ranges of the function from the table. Students learn how to develop an explicit or recursive formula given a sequence of integers. They explore many examples of functions, including sequences. Students interpret functions presented graphically, numerically, symbolically, and verbally. They translate between representations and understand the limitations of various representations. They interpret arithmetic sequences as linear functions. In Algebra I, students must compare and contrast relationships between two functions; these skills will be used...
throughout Algebra I. Understanding the concept of a function and using function notation is a major standard in Algebra I according to the PARCC Model Content Frameworks.

**Future Learning**

Students will study quadratic and exponential functions later in Algebra I. In Algebra II, students will extend their knowledge of building a function that models a relationship between higher order polynomial functions. Later in Algebra I, students will use their knowledge of arithmetic sequences to continue their study of geometric sequences. Precalculus and Calculus students will use arithmetic sequences and series in their studies.

**Additional Findings**

Students have difficulty understanding what the solution means in the context of the problem. This can be clarified by connecting different representations of the problem. “Students translate among verbal, tabular, graphical, and algebraic representations of functions, and they describe how such aspects of a function as slope and y-intercept appear in different representations.” *(Curriculum Focal Points, p. 20)*

**Assessment**

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Show how to determine if a sequence represents a function.
- Model real world problems using arithmetic sequences.
- Identify the domain and range of a function
- Determine whether a relation is a function
- Find values for a function.
- Interpret functions using real world problems
- Use a graphing calculator to represent functions as a graph and as a table
- Use function notation to evaluate and interpret the results of solving word problems
- Write a function that describes a proportional relationship
- Write a function that describes a non proportional relationship
Instruction

Learning Objectives

Students will be able to:

- Recognize that sequences are functions.
- Represent and interpret graphs as relations
- Determine whether a relation is a function
- Find the values for the function.
- Use technology to represent a function as a graph and as a table
- Use function notation to evaluate and interpret the results of solving word problems
- Write a function that describes a quantitative relationship
- Demonstrate understanding of concepts and skills learned in this unit

Resources

In *Algebra 1*, (Glencoe McGraw Hill, 2010)

- Section 1.6 pp. 38 to 44
- Section 1.7 pp. 45 to 52
- Section 1.7 Graphing Technology Lab. P. 53
- Section 3.5 pp. 187 to 193, 204
- Section 3.6 pp. 195 to 200
- *Chapter 1 Resource Masters* of corresponding sections
- *Chapter 3 Resource Masters* of corresponding sections
- *5-Minute Check Transparencies* for corresponding sections
- Interactive Classroom CD - PowerPoint presentations
- Teacher Works Plus CD-ROM
- Teaching with Foldables (Dinah Zike; Glencoe McGraw Hill 2010)

*Quick Review Math Handbook* (Glencoe McGraw Hill) 2010

- Sections 6.7 (p. 291 Arithmetic Sequences)

Exam View Assessment Suite Software

Math Online, glencoe.com

*Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.*
Materials
Graphing calculators

Instructional Considerations

Key Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>domain</td>
<td>function</td>
</tr>
<tr>
<td>range (with relation to function)</td>
<td>vertical line test</td>
</tr>
<tr>
<td>mapping</td>
<td>continuous function</td>
</tr>
<tr>
<td>dependent variable</td>
<td>discrete function</td>
</tr>
<tr>
<td>independent variable</td>
<td>common difference</td>
</tr>
<tr>
<td>arithmetic sequence</td>
<td></td>
</tr>
</tbody>
</table>

Planning for Effective Instructional Design and Delivery

The reinforced vocabulary taught in previous grades or units includes terms, sequences, expressions, coordinate system, x and y axes, origin, and ordered pair.

The focus of this unit is writing arithmetic sequences as linear functions. The work in unit 1.2 dealing with expressions provides significant support for student work with functions.

Show students there are four different ways to display functions: table, mapping diagram, equation, and graph. Explain to them that there are two variables (x and y), where the x-values are the domain and the y-values are the range of the function. To help students understand input and output of functions, give them a number and tell them it is their x-coordinate, or input. Then give them instructions to add, subtract, multiply or divide. Their result will be the y-coordinate or output. Next have them place their points on a graph and to see the function. Have students use a graphic organizer to identify advantages and disadvantages of each type function displays (table, mapping, equation, graph).

The Graphing Technology Lab for section 1.7 allows students to see the relationship between the table and graph of a function. A cooperative grouping model such as grouping students of mixed abilities would work well here. Students could help one another with equations which would allow the instructor to better monitor the groups and focus on the students with the greatest needs. Section 3-6 extends students’ work from 1-7 and provides opportunities for students to write functions from a table and graph. Students could use the Ti-nSpire calculator to check their work and create functions of their own to model.

The “Why” introduction of section 3.5 is a good starting point to develop student awareness of patterns. Make sure students understand that distance increases with time. Students must also be able to model sequences context. You may want to use Real World Example 4(P.190) with students. Students should have the opportunity to create and design sequence problems that lead to modeling with functions.

To develop conceptual understanding of sequences, students can model the sequences with algebra tiles. This will help students see the common difference and write a function to find the nth term.

Have students demonstrate their learning of arithmetic sequences by arranging them into cooperative groups, assign each group a sequence, have them write how they found the nth term for the sequence and...
resulting function. Have the groups post their work on chart paper and do a gallery walk so that students can comment on the other groups’ work.

Incorporate the Essential Questions as part of the daily lesson. Options include using them as a “do now” to activate prior knowledge of the previous day’s lesson, using them as an exit ticket by having students respond to it and post it, or hand it in as they exit the classroom, or using them as other formative assessments. Essential questions should be included in the unit assessment.

The 5-minute check transparencies can be used as a cue, questions, and advance organizers strategy as students will be activating prior knowledge. Some 5-minute checks may take longer than the allotted time, so consider choosing only problems that activate prior knowledge and use the rest for differentiation, to formatively assess student learning, as an exit ticket, or assigning for homework.

For planning considerations read through the teacher edition for suggestions about scaffolding techniques, using additional examples, and differentiated instructional guidelines as suggested by the Glencoe resource.

You may use the transparencies provided in the ancillary material as focus activities, review, or an exit activity. Additional activities and examples may be used for homework assignments.