Algebra II, Quarter 1, Unit 1.1

Parent Functions and Transformations

Overview

Number of instruction days: 5–7 (1 day = 53 minutes)

Content to Be Learned

- Recognize the properties and characteristics of families of functions.
- Identify the effect on a graph of one or more transformations of a given function.
- Identify common effects of each transformation across function types.

Mathematical Practices to Be Integrated

5 Use appropriate tools strategically.
- Use graph paper and patty paper to explore and analyze the effects of parameter changes to various parent functions.
- Use the TI-Nspire calculators to observe the resulting change in the graph of a function as the constants are altered.

7 Look for and make use of structure.
- Recognize the common effects that each parameter change has on the different parent functions.
- Use structure of functions to compare the transformed graphs to the parent graphs.

Essential Questions

- What are the similarities and differences among transformations of all the parent functions?
- Why is it important to study transformations of functions?
- What are the different effects of changing the parameters on a parent function?
- How are transformations used in electronic games and/or animation?
Common Core State Standards for Mathematical Content

Functions

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<th>Building Functions</th>
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Build new functions from existing functions \([\text{Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types}]\)

F-BF.3 Identify the effect on the graph of replacing \(f(x)\) by \(f(x) + k\), \(k f(x)\), \(f(kx)\), and \(f(x + k)\) for specific values of \(k\) (both positive and negative); find the value of \(k\) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see \(7 \times 8\) equals the well remembered \(7 \times 5 + 7 \times 3\), in preparation for learning about the distributive
property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**Clarifying the Standards**

**Prior Learning**

Since Grade 6, students have represented relationships between independent and dependent variables on the coordinate plane. In Grade 8, students qualitatively described function relationships, learned the definition of a function, and represented input and output on the coordinate plane. They also compared properties of functions. In Algebra I, students built functions and compared linear, exponential, and quadratic functions. They also examined the effect of four transformations on linear, exponential, quadratic, and absolute value functions. In Geometry, students described transformations as functions that take points in the plane as inputs and give other points as outputs.

**Current Learning**

In this unit, students study the properties and characteristics of the family of functions. They experiment with different types of transformations and their effects on a single graph, as well as the common effects of each transformation across different function types. Building new functions from existing functions is classified as additional content in Algebra II according to the PARCC Model Content Frameworks.

**Future Learning**

In Precalculus, students will continue to explore the effects of each transformation on different types of functions, including trigonometric functions.

**Additional Findings**

*Developing Essential Understanding of Functions for Teaching Mathematics in Grades 9-12* identifies five “big ideas” essential to understanding functions. According to Big Idea 1, the concept of function is intentionally broad and flexible, allowing it to apply to a wide range of situations. According to Big Idea 2, functions provide a means to describe how related quantities vary together.

According to *Principles and Standards for School Mathematics*, students should use technology tools to represent and study the behavior of polynomial, exponential, rational, and periodic functions among
others; they will learn to combine functions, express them in equivalent forms, compose them, and find inverses where possible (p. 297). As students do so, they will learn to recognize the characteristics of various classes of functions.

Assessment

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Identify families of functions using their properties and characteristics.
- Describe the effect that a transformation has on the graph of a given function.
- Identify transformations of parent function.
- Recognize odd and even functions from their graphs and symbolic representations
- Graph functions under the given transformations:
  - Reflection,
  - Translation,
  - Dilation.
- Describe similarities and differences among transformations of parent functions.
Instruction

Learning Objectives

Students will be able to:

- Graph, identify, and predict transformations of parent functions using the graphing calculator, including linear, absolute value, piecewise, step, quadratic, rational, radical, exponential, logarithmic, and polynomial functions.
- Reflect on and demonstrate understanding of parent functions and transformations.

Resources


- Section 2-7 Parent Function and Transformations (pp. 109-116)
- Graphing Calculator Exercise *Families of Lines*: Section 2-7
- Chapter 2 Resources Masters (pp. 43 - 48)
- Study Notebook (pp. 31 – 32)
- Interactive Classroom CD (PowerPoint Presentations)
- Teacher Works CD ROM
- Exam View Assessment Suite

**TI-Nspire Teacher Software**

*Families of Functions* can be found at education.ti.com. See the Supplementary Unit Materials section of this binder for the student and teacher notes for these activities.

*Transformations: Translating Functions* can be found at education.ti.com. See the Supplementary Materials Section of this Binder for the Student and Teacher Notes for This Activity.

**The district-created Transformation of a Graph worksheet** is also in the Supplementary Unit Materials section.
Materials

TI-Nspire graphing calculators, graph paper, patty paper, ruler

Instructional Considerations

Key Vocabulary

amplitude  stretch

compression

Planning for Effective Instructional Design and Delivery

Reinforced vocabulary taught in previous grades or units: cubic, exponential, linear, parabola, polynomial, quadratic, rotation, transformation, and translation.

Use the posted Essential Questions at the end of each lesson objective as a prompt to ask the students which question was worked on that particular day. You can use Essential Questions for discussion or as an exit ticket. The Essential Questions are an excellent tool for vocabulary building and can also be used as an oral form of formative assessment.

This unit has been added to give Algebra II students an overview of the functions they will be working with throughout the course. Students should have a conceptual understanding of the transformation of all functions by using a calculator in their study. In the Families of Functions activity available on education.ti.com, students change the values for a, h, and k and observe the effects on the graphs. The types of functions included in this activity are: quadratic, absolute value, exponential, logarithmic, cubic, and sine functions. Additionally, the Transformations: Translating Functions activity provide the opportunity for students to translate a function by adding a constant and write the appropriate symbolic representation for the translated function. Students horizontally translate a function by adding a constant and write the appropriate symbolic representation for the translated function. Finally, they identify the effect of a and b in \( y = f(x + a) + b \) on the graph of a general function \( y = f(x) \).

See the district-created worksheet listed in the materials section for some activities to do with students in this unit.

Have students graph each set of the functions below on one graph and answer the questions.

Set 1

What characteristics make the following functions quadratic functions?
What are the domain and range of the parent function?

a. \( y = x^2 \)

b. \( y = \frac{1}{8} x^2 \)

c. \( y = 3x^2 \)

d. \( y = -3x^2 \)

Set 2

Give the name of the functions in this set.

What characteristics caused you to give the functions that name?

What are the domain and range of the functions?

a. \( f(x) = |x| \)

b. \( f(x) = |x| + 2 \)

c. \( f(x) = |x| - 3 \)

d. \( f(x) = -|x| + 4 \)

e. What transformation occurred in the functions above?

Set 3

Give the name of the functions in this set.

What characteristics caused you to give the functions that name?

What are the domain and range of the functions?

a. \( f(x) = \sqrt{x} \)

b. \( f(x) = \sqrt{x - 1} \)

c. \( f(x) = \sqrt{x + 2} \)

d. \( f(x) = -\sqrt{x + 4} \)

e. What transformation occurred in the functions above?
Students can apply their understanding of horizontal and vertical compression/dilation of the amplitude of each of the functions by identifying similarities and differences in the examples given above. Ask students to compare the functions in the examples and identify the characteristics that are the same or different when using the same transformation. Help students think about the similarities and differences by asking questions such as the following: What does the degree of each polynomial mean? Which functions are similar, and which are different? Are the domain and the range the same for the quadratic and absolute value parent functions?

In a lesson to explore the graphs of polynomial functions, students may use the nonlinguistic representations strategy of pictographic representations to understand the characteristics of the function.

To help ascertain if students have a clear understanding of polynomial characteristics, put them into groups and give each group one of the problems below. Each group will create a table of values (either by substituting appropriate domain values given the situation or by using the table feature of the graphing calculator).

Students will be responsible for the following information at the end of their exploration.

- Identify the following on the graph: the roots of the function, the intervals of the function, and the x- and y-intercepts.
- Explain the following: the similarities of zeros, roots, solutions, and x-intercepts; rates of change between the intervals; all local maximum and minimum points; and the turning points of the graph and its end behavior.
- Justify the following: an appropriate domain and range of the function and their answers to the questions of their given problem.