Mathematical Models with Applications, Quarter 4, Unit 4.2

Applications of Polynomials

Overview

Number of instruction days: 3 – 5 (1 day = 53 minutes)

Content to Be Learned

- Understand that polynomials form a system similar to integers (closed under addition, subtraction, and multiplication).
- Interpret terms, factors, and coefficients of polynomial expressions in their context.
- Use the characteristics and structure of an expression to recognize ways to rewrite it.
- Apply and analyze polynomial functions using real-world scenarios.

Mathematical Practices to Be Integrated

4 Model with mathematics.

- Model with mathematics to solve real world problems involving operations with polynomials.

7 Look for and make use of structure

- Rewrite polynomial expressions in equivalent forms.
- Recognize structural similarities between integers and polynomials.

Essential Questions

- How do you simplify polynomial expressions?
- How are polynomials used to solve area or volume problems?
- Why is it important to rewrite polynomial expressions in different forms?
Standards

Common Core State Standards for Mathematical Content

Algebra

Seeing Structure in Expressions  A-SSE

Interpret the structure of expressions [Polynomial and rational]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

Arithmetic with Polynomials and Rational Expressions  A-APR

Perform arithmetic operations on polynomials [Linear and quadratic]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In
the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

### Clarifying the Standards

**Prior Learning**

Students have studied the distributive property since the third grade, when they used area models to multiply numbers. Students deepened their understanding of the distributive property in the seventh grade, when they solved equations of the form $p(x+q) = r$. In eighth grade students learned and applied properties of integer exponents. In Algebra 1, students extended the properties of exponents to rational exponents. Students performed arithmetic operations on linear and quadratic polynomials and factored quadratic expressions. They also performed arithmetic operations (addition, subtraction, multiplication) on linear and quadratic polynomials. They rewrote linear, quadratic, and exponential expressions in different but equivalent forms.

**Current Learning**

In this unit, students develop an understanding of the structure and operations of polynomials through real world applications. They add, subtract, and multiply polynomials within these scenarios.

**Future Learning**

Operations with polynomials will be a critical area in Algebra 2. Students will investigate the structural similarities between polynomials and integers. They will add, subtract, multiply, and divide polynomials. Students will prove and use polynomial identities to describe numerical relationships and will write polynomial expressions in different forms. Students will also learn and apply the Binomial Theorem. They will study polynomials of a degree higher than quadratic and connect division of polynomials with long division of integers. Students will identify zeros of polynomials, including complex zeros of quadratic polynomials, and will make connections between zeros of polynomials and solutions of polynomial equations. They will understand and apply the Remainder Theorem. In Pre-Calculus, students will perform arithmetic operations with complex numbers. They will also represent complex numbers and their operations on the complex plane.

**Additional Findings**

No additional findings.
Assessment

When constructing an end of unit assessment, be aware that the assessment should measure your students’ understanding of the “big ideas” indicated within the standards. The CCSS Content Standards and the CCSS Practice Standards should be considered when designing assessments. Standards based mathematics assessment items should vary in difficulty, content and type. The assessment should include a mix of items which could include multiple choice items, short and extended response items and performance based tasks. When creating your assessment you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

Math Models students should be provided with multiple, alternative methods to express their understandings of the concepts that follow:

- Simplify polynomial expressions, involving powers, using the closure property of polynomials, to create equivalent expressions.
- Simplify polynomial expressions using addition, subtraction, and multiplication.
- Identify the different parts of polynomial expressions and explain their meaning within the context of a problem.
- Rewrite polynomial expressions in different equivalent forms by combining like terms.

Instruction

Learning Objectives

Students will be able to:

- Reinforce the closure property of polynomials by adding, subtracting, multiplying, and simplifying polynomial expressions involving powers.
• Generate equivalent polynomials expressions to solve problems.
• Identify parts of polynomial expressions, such as terms, factors, and coefficients in the context of problems.
• Demonstrate understanding of concepts and skills learned in this unit.

Resources
• Additional Resources located in the Supplementary Unit Materials Section of the Binder:
  o TEXTEAMS Algebraic and Geometric Modeling Institute, 2002
    ▪ III. Modeling and Money
      o Activity: Downsizing (p. 216)
      o Activity: Reflect and Apply (p. 217)
  o Multiplying Using the Box Method – Teacher
  o Multiplying Using the Box Method – Student
  o CK-12 Study Guides - Operations with Polynomials

Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery section below for specific recommendations.

Materials

TI-Nspire Graphing Calculator, Algebra tiles, gridded chart paper, markers, glue, construction paper and sentence strips.

Instructional Considerations

Key Vocabulary
No new vocabulary.

Planning for Effective Instructional Design and Delivery

Reinforced vocabulary from previous grades or units: degree, monomial, polynomial, terms, degree of a polynomial, trinomial, binomial, distributive property and leading coefficient.

This unit is composed exclusively of supplementary materials.

As an introductory activity, have the class discuss the following: Given $4x^2 - x$, identify the leading coefficient, degree, and base. Explain what the degree tells you about possible solutions. Then break
students into groups of two or three. Give each group of students two different polynomials (i.e., $3x^4 + 4x^2 + x - 3$ and $3x^2 - 3$) written on large strips of paper. Individually, students should compare the two polynomials to help them decide what similarities and differences are present. Students then write the similarities and differences on individual sentence strips. Then have the group members discuss their strips together. When they agree on what they consider to be the accurate and complete list of similarities and differences, have students place their strips into a Venn diagram. Groups will share their finished task. Each group will evaluate the other groups’ work.

Use a graphic organizer or game-type activity to access prior knowledge regarding polynomials and operations with polynomial expressions. Consider differentiating instruction by having students play the Algebra Battleship for Polynomials – Adding and Subtracting on the following website: [http://www.quia.com/ba/28820.html](http://www.quia.com/ba/28820.html). You will need to verify that the website is accessible prior to the setting it up for your students.

This unit develops the structural similarities between polynomials and integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property.

**Nonlinguistic representations** including graphic organizers and physical models such as algebra tiles would be appropriate for teaching operations with polynomials. Algebra tiles provide conceptual understanding of combining like terms. Algebra tiles and area models also help students understand the distributive property through manipulating the tiles and help to identify strategies to write expressions in different ways. For example:

\[
\begin{array}{c|c|c}
-3 & x & 7 \\
\hline
\end{array} =
\begin{array}{c|c|c}
-3(x) & -3(7) \\
\hline
-3x & -21 \\
\end{array}
\]

Use a graphic organizer (nonlinguistic representation) such as Multiplying Using the Box Method to differentiate instruction for multiplying polynomials. The teacher and student notes for this activity are located in the Supplementary Unit Materials Section of this binder.

The focus of this unit is applying polynomials in real world situations. The *Downsizing Activity* located in the Supplementary Unit Materials Section of this binder provides the opportunity to make connections between real world and abstract theory of polynomials.

Additional resources for polynomials can be found on CK-12 book using the following website: [http://www.ck12.org/flexbook/](http://www.ck12.org/flexbook/). A CK-12 Study Guide for Operations with Polynomials has been included in the Supplementary Unit Materials Section of this binder. The CK-12 Algebra 1, SE provides problems involving application of polynomials using area and perimeter of two-dimensional figures. The CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Teachers are able use the FlexBook Platform™ on the
website to create resources aligned with their standards and units. All resources on CK-12 are available for free.